The Vector Wave Propagation Method - VWPM

An extension to the scalar wave propagation method - examples and correlation to RCWA

Matthias Fertig et. al.

The scalar wave propagation method (WPM) [1] has removed the limitations of the beam propagation method (BPM) but does not consider full three-dimensional vector fields. The vector wave propagation method (VWPM) extends the range of applications to systems which strongly depend on the polarization of the wave (i.e. large aperture lenses or high frequency diffractive gratings). The new approach is valid for propagation angles up to 85 degrees and utilizes the Fresnel coefficients at system boundaries.

Given: system $\mathcal{N} \in \mathcal{C}^{N_x \times N_y \times N_z}$ with
refractive index $\hat{n}_m = n_m(\mathbf{r}_\perp, z_m) + i\kappa_m(\mathbf{r}_\perp, z_m)$
in the m[th] layer

Result: field distribution $\mathrm{E}^{(3)}(\mathbf{r})$

Theory:

1.Plane wave decomposition

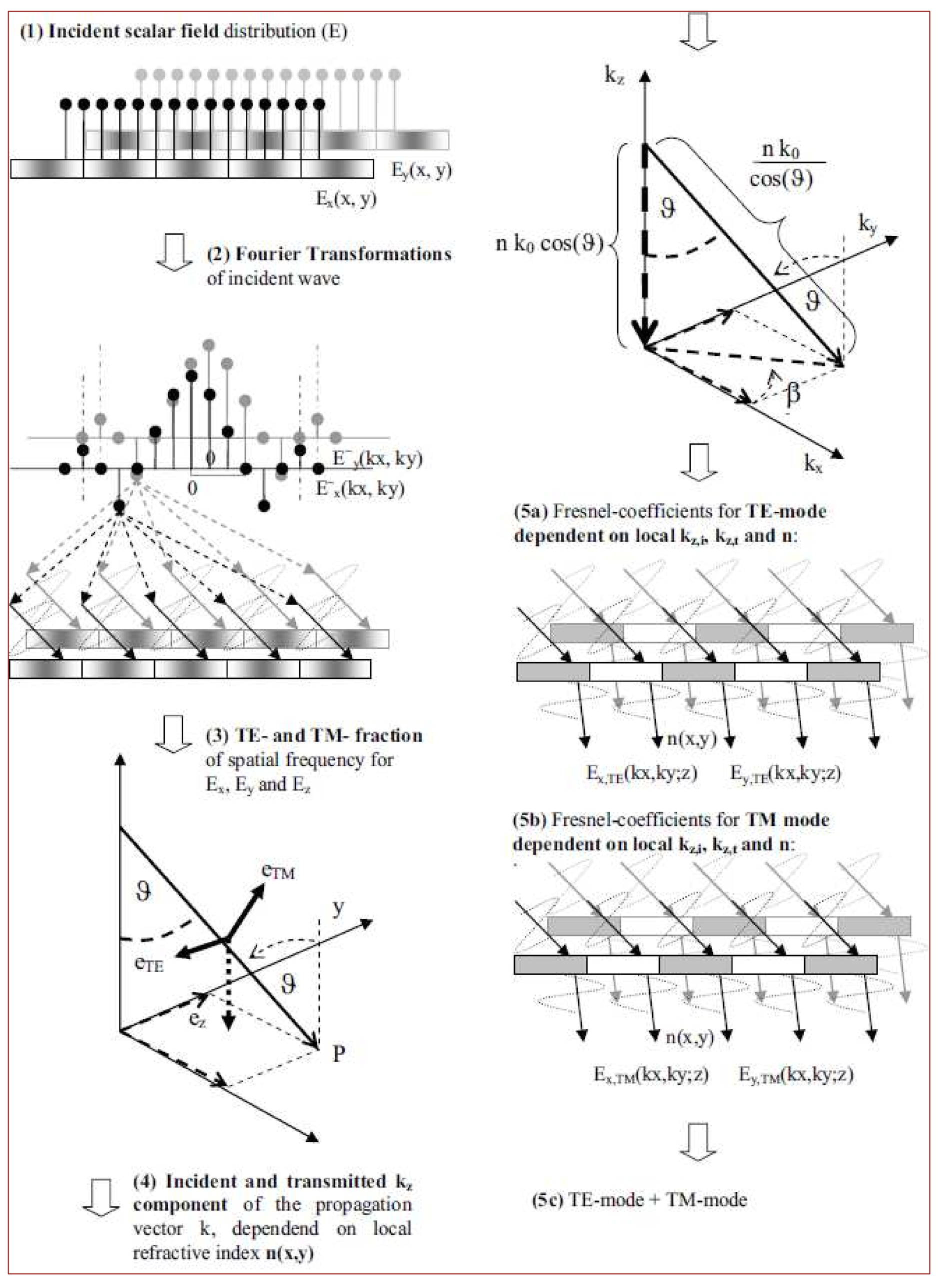
$$\tilde{\mathbf{E}}_{\perp,m}(\mathbf{k}_\perp) = \iint \mathbf{E}_{\perp,m}(\mathbf{r}_\perp) \cdot \exp(-i\,\mathbf{k}_\perp \mathbf{r}_\perp)\, d^2\mathbf{r}_\perp$$

2.Transfer at interface

$$\tilde{\mathbf{E}}_{\perp,m+1}(\mathbf{k}_\perp) = \mathbf{M}_{m,m+1}(\mathbf{r}_\perp, \mathbf{k}_\perp) \cdot \tilde{\mathbf{E}}_{\perp,m}(\mathbf{k}_\perp)$$

3. Propagation inside the inhomogeneous layer

$$\boldsymbol{\varepsilon}_{\perp,m+1}(\mathbf{r}_\perp, \mathbf{k}_\perp) = \tilde{\mathbf{E}}_{\perp,m+1}(\mathbf{k}_\perp) \cdot \exp\left(i\phi_{m+1}(\mathbf{r}_\perp, \mathbf{k}_\perp)\right)$$



$$\boldsymbol{\varepsilon}_z = -\frac{\left(\boldsymbol{\varepsilon}_x k_x + \boldsymbol{\varepsilon}_y k_y\right)}{k_z} \qquad \textit{Transversality}$$

4. Summation

$$\mathbf{E}_{m+1}(\mathbf{r}_\perp) = \iint \boldsymbol{\varepsilon}_{m+1}(\mathbf{r}_\perp, \mathbf{k}_\perp) \cdot \exp(i\,\mathbf{k}_\perp \mathbf{r}_\perp) \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}$$

Transfer-Matrix with Fresnel coeff. t_s, t_p:

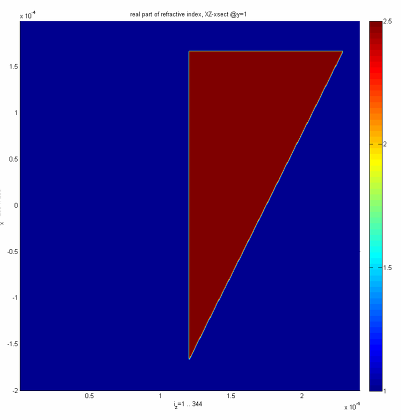
$$\mathbf{M}_{m,m+1} = \frac{1}{k_\perp^2} \begin{pmatrix} k_y^2 t_s + k_x^2 \hat{t}_p & k_x k_y (\hat{t}_p - t_s) \\ k_x k_y (\hat{t}_p - t_s) & k_x^2 t_s + k_y^2 \hat{t}_p \end{pmatrix}$$

$$\hat{t}_p = \frac{n_m k_{z,m+1}}{n_{m+1} k_{z,m}} t_p$$

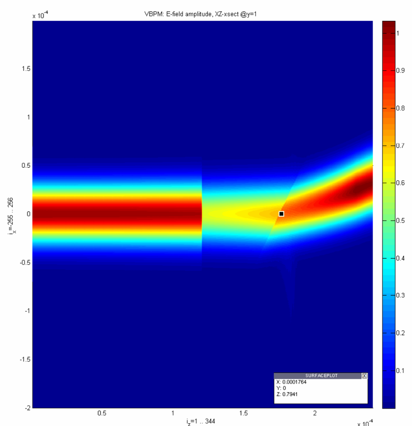
Propagation phase :

$$\phi_m(\mathbf{r}_\perp, \mathbf{k}_\perp) = dz\sqrt{n_m(\mathbf{r}_\perp)^2 k_0^2 - k_\perp^2}$$

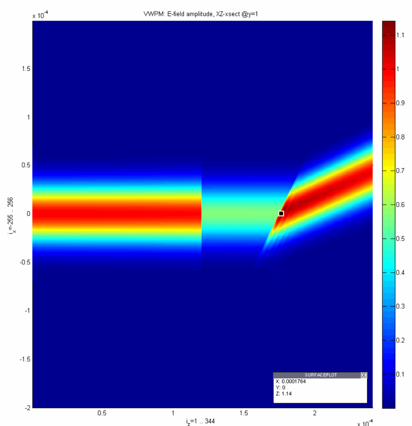
1. 2D simulation of a prism - traversed by a TE-polarized plane wave



Prism scene: imaginary part of refractive index is zero.

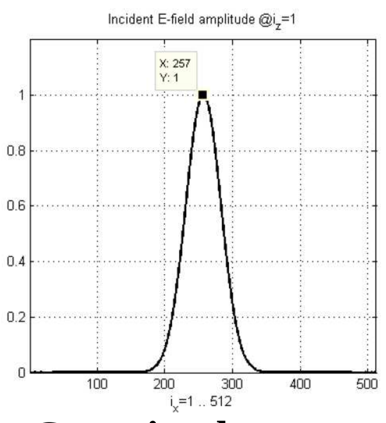


1.a. VBPM: increasing amplitude inside the prism.

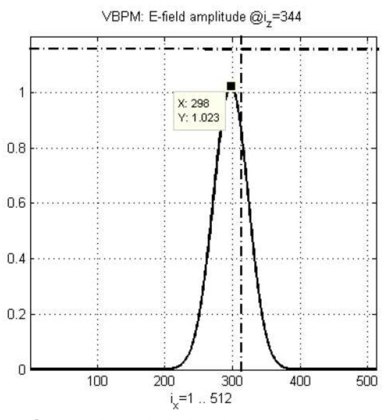


1.b. VWPM: constant amplitude inside prism.

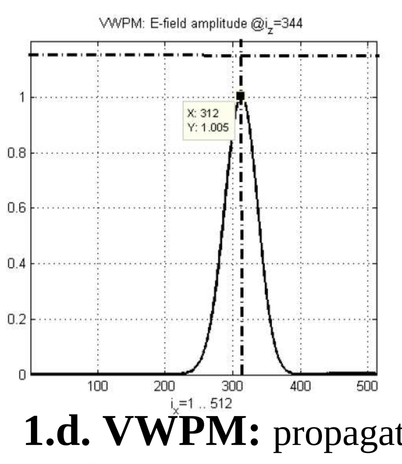
Fresnel coefficients
The VBPM uses the averaged refractive index and therefore shows an increasing amplitude inside the prism. The amplitude of the VWPM deviates 4% from theory and is thus 10 times as accurate.



Gaussian beam



1.c. VBPM: propagated amplitude

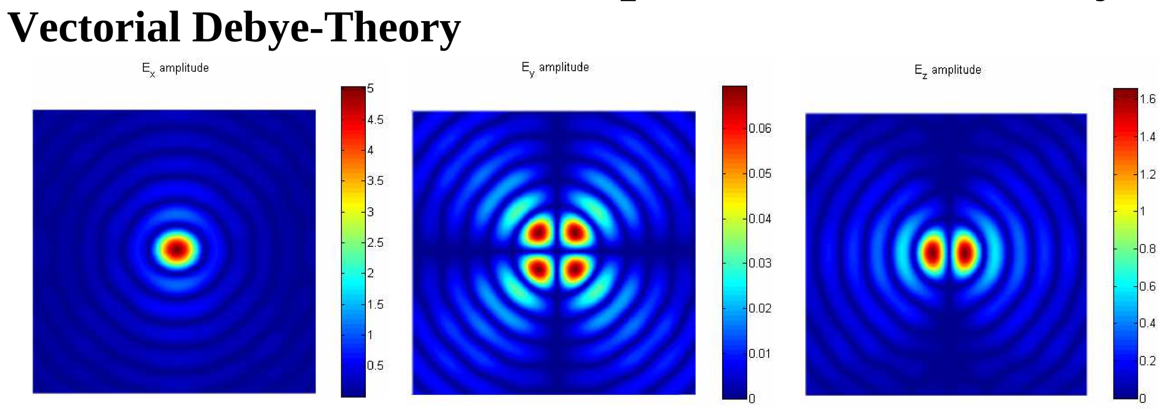


1.d. VWPM: propagated amplitude

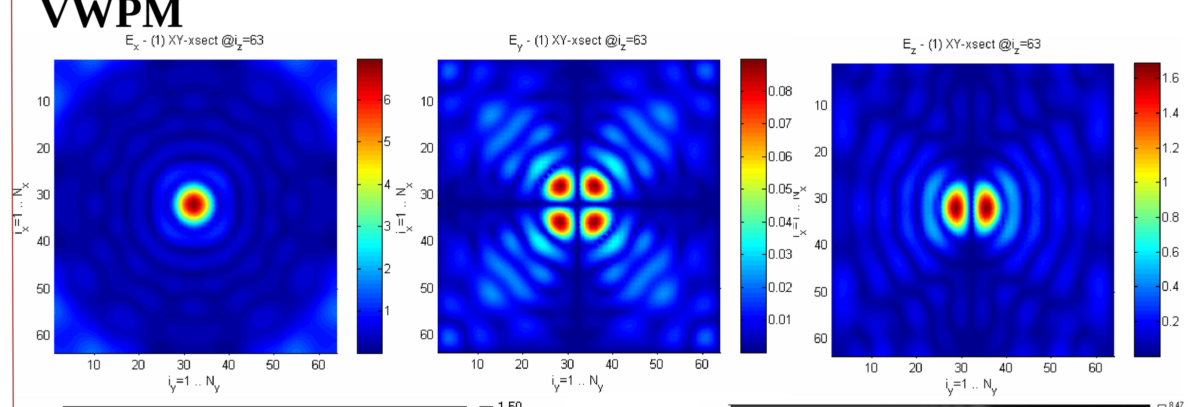
Refraction angle
The VWPM shows perfect beam location compared to the VBPM. The location of the Gaussian peak matches with Snell's law.

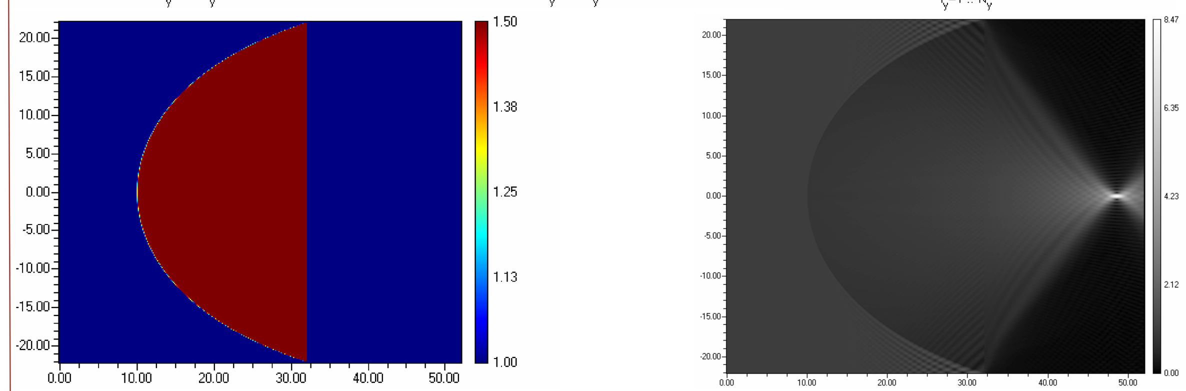
2. 3D simulation of an asphere - traversed by a x-polarized plane wave

Vectorial Debye-Theory



VWPM





2.a. Vector field verification from vectorial Debye theory

Vectorial Debye-Integral:

$$\mathrm{E}(\mathrm{r}_\perp) = -E_0 \frac{in}{\lambda} \iint \mathrm{M}_e(\mathrm{s}_\perp) \frac{\tilde{\mathrm{E}}(\mathrm{s}_\perp)}{s_z} \delta s_\perp$$

2.b. Vector field components simulated with the VWPM. The simulation shows an acceptable match of the amplitude in the focus of a perfect asphere.

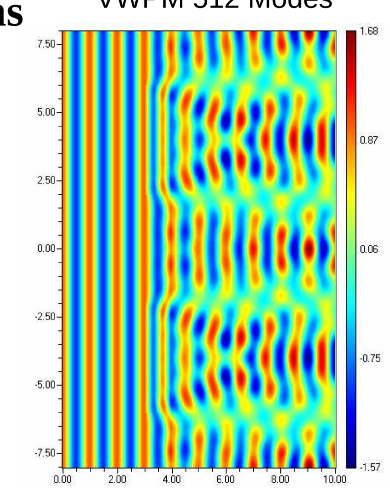
2.c. x-z-Section
The VWPM shows a diffraction limited, unaberrated spot, as expected from a perfect aspheric focussing lens (left). The focus location agrees perfectly with ray tracing. The BPM produces a longitudinal focal shift and a significant spherical aberration.

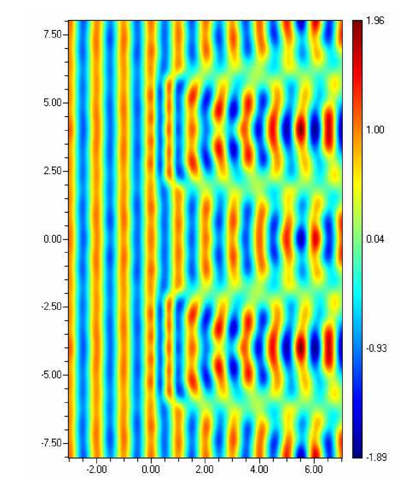
3. Simulation of a grating

3.a. Grating dimensions

Phase Ronchi
$\lambda = 1\mu m$
$h = 1\lambda$
$p_x = p_y = 8\lambda$
$E_{in} = \mathbf{e}_x$

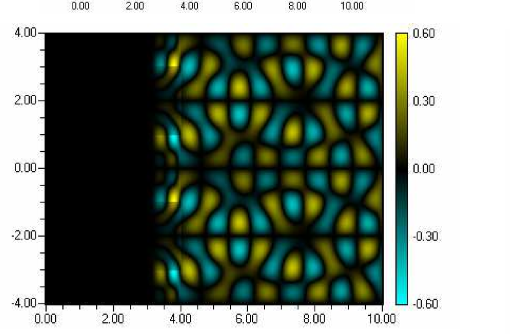
E_x

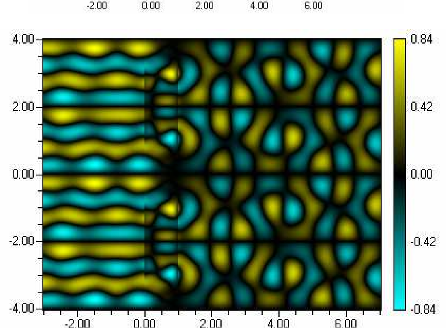




$\lambda = 1\mu m$
$h = 1\lambda$
$p_x = p_y = 4\lambda$
$E_{in} = \mathbf{e}_x$

E_z





3.b. Runtime

VWPM-512 2D:	2 sec.
VWPM-64 3D:	17 sec.
VWPM-128 3D:	6.5 min.

3.c. Complexity

RCWA:	$O(N^4)$
VWPM:	$O(N^2)$
BPM:	$O(N \log(N))$

Poynting vector and intensity

The local distribution of the intensity is derived from the electric field.
The magnetic field vector follows from Maxwell

$$\tilde{\mathrm{H}} = \sqrt{\frac{\epsilon\epsilon_0}{\mu\mu_0}} \left(\frac{\mathrm{k}}{k} \times \mathrm{E}\right)$$

Three-dimensional magnetic vector derived from transversal components of the electric field

$$\tilde{\mathrm{H}}^{(3)}(\mathrm{r}_\perp, \mathrm{k}) = \mathrm{T}(\mathrm{r}_\perp, \mathrm{k}) \cdot \tilde{\mathrm{E}}_\perp \qquad \mathrm{T}(\mathrm{r}_\perp, \mathrm{k}) = \frac{1}{k \cdot k_z} \cdot \sqrt{\frac{\epsilon\epsilon_0}{\mu\mu_0}} \begin{pmatrix} -k_x k_y & -(k^2 - k_x^2) \\ (k^2 - k_y^2) & k_x k_y \\ -k_y k_z & k_x k_z \end{pmatrix}$$

The summation of the magnetic wave components over the aperture yields the distribution of the magnetic field in the aperture and the Poynting vector is then

$$\mathrm{S} = \frac{1}{2} Re\left(\mathrm{E}^{(3)}(\mathrm{r}) \times \mathrm{H}^{(3)*}(\mathrm{r})\right)$$

The local intensity is

$$I(\mathrm{r}) = \langle |\mathrm{S}(\mathrm{r})| \rangle_t = \frac{n(\mathrm{r})}{2Z_0} \mathrm{E}^{(3)}(\mathrm{r}) \mathrm{E}^{(3)*}(\mathrm{r})$$

Conclusion: The VWPM enables calculation of the propagation of a vector wave through an inhomogeneous three-dimensional system. The simulations show that the method is accurate also for non-paraxial propagation and is not limited to small index variations. Apart from reflections, the VWPM is exact. The computational effort is significantly smaller than rigorous methods and the VWPM provides efficient ways for parallelization. In homogeneous regions the VWPM agrees fully with the vector plane wave decomposition, which has been used for speed-up.

[1] K.-H. Brenner and W. Singer, *Light propagation through microlens: a new simulation method* in Applied Optics 32, (1993), 4984.4988.
[2] M. Feit, J. Fleck, *Light propagation in graded index fibres*, in Applied Optics, (1978), 3390-3998.

Contact:
Prof. Dr. Matthias Fertig
matthias.fertig@htwg-konstanz.de
Braunegger Straße 55
78462 Konstanz