Session 4

Exercise 1: Focal length

Given that the focal length of a lens n in air $n_a = 1$ is f. Show that the focal length of the same lens in water n_w is

$$f_w = \frac{n_w(n-1)}{n_w - n} f$$

Hints: Use the formula for f_o given in the course material.

Exercise 2: Magnification

To obtain a magnification M, the object distance s_o needs to be

$$s_o = \frac{M+1}{M} f$$

a) Derive the equation.

b) A camera with a 50 mm focal length is used to photograph a person 1.75 m tall. Derive the object distance s_o to get an image of size 24 mm.

Hint: Use the imaging law and let $s_i = -M s_o$.

Exercise 3: Focal point and refractive index

a) Show that a change in the refractive index dn induces a change in the focal point df described by the relation

$$\frac{df}{f} = -\frac{dn}{n-1}$$

b) Use the relation to calculate the focal length of a lens for blue light ($n = 1.53$), if 20 cm is the focal length for red light $n = 1.47$.

Hint: Use the formula for the focal point of thin lenses and perform the first derivative of df/dn. Then rearrange the terms.

Exercise 4: Mirror Formula

Show that a convex mirror cannot create a real image.

Hints: Use the Mirror Formula from the course material and show that the image distance is negative if the object distance is positive.

Exercise 5: Fermats principle and Gaussian Optics

The expression

$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

is obtained from Fermats Principle, i.e. $dOPL/d\phi = 0$.

a) Show that the expression is obtained from $OPL = n_1 l_o(\phi) + n_2 l_i(\phi)$

b) Show that for small angles, where $l_i \approx s_i$ and $l_o \approx s_o$, the expression

$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{n_2 - n_1}{R}$$

for Gaussian Optics is obtained.

Hints: Script modul 4 pages 11-14.

Exercise 6: MATLAB exercise: ABCD-matrix

Use the ABCD-matrix for the following exercises.

a) Write a set of Matlab functions to return the ABCD-matrix for
a.1) propagation through homogeneous medium
a.2) transformation of rays at plane interfaces
with a refractive index ni and nt
a.3) transformation of rays at spherical interfaces
with a radius R and a refractive index ni and nt
a.4) transformation of rays through thin plane-convex lenses
with a radius R and a refractive index nl and nm
a.5) transformation of rays through thick lenses
with a radius Rl, Rr and a thickness D.

b) Write a Matlab program to plot the transformation of rays at a plane interface. Calculate the deviation in the paraxial approximation $\tan\theta = \theta$ in percent?

c) Referring to the results in b), up to what angle of incidence is the deviation in the paraxial approximation below three percent for $ni = 1$ and $nt = 3.4$? Get the result from simulations and give the analytic term.

d) Referring to the results in c), how would the deviation change for $ni = 3.4$ and $nt = 1$?

e) Extend the Matlab program to show the propagation for a configurable set of parallel rays through a thin lens with focal length $f = 5\ cm$. Show by simulation how the image point shifts in the focal plane for a variation of the angle of incidence and how an image point is formed from object points.